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Research Report CCS 407

THE MEDIAN ROUTING STUDY METHOD APPLIED TO THE 1947 CONDITION OF THE PECOS RIVER COMPACT

by

A. Charnes
R. Dickinson*
S. Duffuaa
J. Heaney*

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*University of Florida

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CENTER FOR CYBERNETIC STUDIES

A. Charnes, Director
Business-Economics Building, 203E
The University of Texas at Austin
Austin, Texas 78712
(512) 471-1821

0. INTRODUCTION

This paper develops a new statistical methodology for implementing the apportionment of the water of the Pecos River between the state of New Mexico and the state of Texas, according to their water compact. The Pecos River is an interstate stream which rises in north central New Mexico and flows in a southerly direction through New Mexico and Texas. The stream flows through semi-aird regions where demand for water exceeds supply. The flow of the river is quite variable (see the area map attached in Figure 1). In this region precipitation ranges from 11 to 14 inches annually. The river joins the Rio Grande near Langtry, Texas. The area of the basin is 35,000 square miles, about 20,000 in New Mexico and the rest in Texas.

The controversy over the allocation of the Pecos River water is more than half a century old. The major concern of this study is to estimate fairly and to allocate fairly the water supply in the river so as to protect development in the two states.

Development on the river has extended over many years. The question of at which stage of development we base the allocation of the water supply has been answered in the Water Compact. It is agreed that this stage of development should be that at the beginning of the year 1947. This is usually referred to in the Compact as the 1947 Condition. The 1947 Condition is the situation in the Pecos River Basin which produced in New Mexico, the man-made depletions from the stage of development existing at the beginning of the year 1947, and the augmented Fort Sumner and Carlsbad acreage.

The major irrigation projects on the Pecos River are the Fort Sumner project of about 5,221 acres and the Carlsbad irrigation district of about 22,368 acres. Both are irrigated by diversion from the river. The augmented

area for Fort Sumner is 6500 acres and for Carlsbad is 25,055 acres. There is a huge pumpage activity between Acme and Artesia at the Roswell area, an irrigated area of 91,744 acres. A detailed description of the river activities is in Section 3.

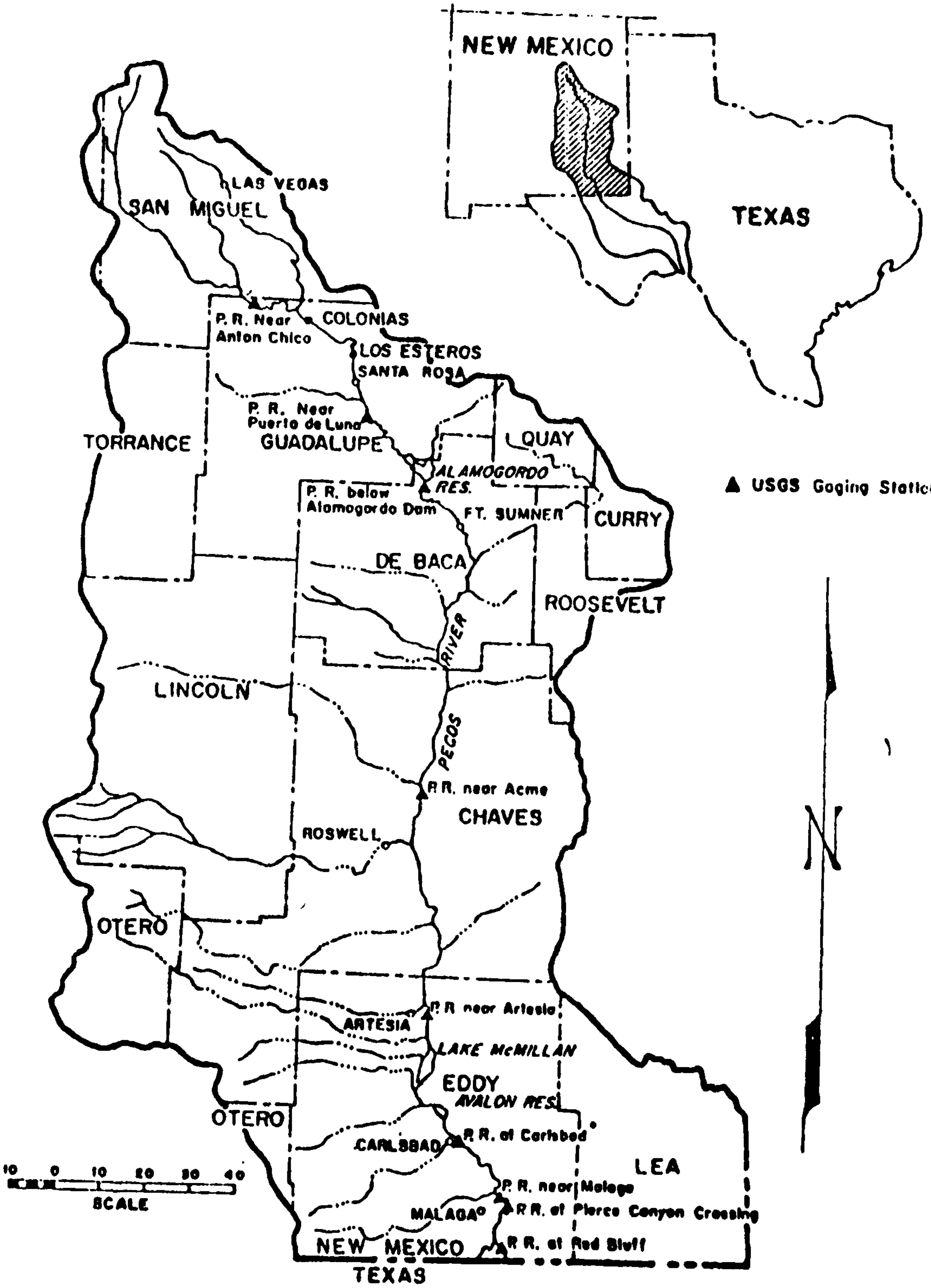
Our problem is to estimate the inflows and outflows in all reaches of the river under the 1947 condition and from that to estimate the delivery obligation of New Mexico to Texas. Earlier methods of routing studies applied to this case resulted in awkward, unreal phenomena such as negative flood inflows. To obviate this and other shortcomings of such routing study methods, our new methodology has been developed.

In the first section a definition of a routing study and the shortcomings of the methodology applied to the Pecos River is given. In addition to that, a summary of the ideas and the aspects of the new methodology for achieving routing studies is outlined. In Section Two the model is formulated in general for a hypothetical river. In Sections Three and Four the case of the Pecos River is addressed in detail, using the new methodology. In the last section we summarize our results and compare them to those of previous studies performed on the river, e.g. to SD109[1/] and RBD[2/].

1/ SD109 is Senate Document 109. It includes the first routing study performed in 1947.

2/ RBD is the "Review of Basic Data" upon which the second routing study in [11] is based.

FIGURE 1



The Pecos River Basin with detail of New Mexico portion.

1. OVERVIEW OF THE MODEL

A routing study is a mathematical model of a river which numerically estimates the availability of water in the river at given points and times under assumed conditions. The water is mathematically passed down stream, considering all depletions and gains in all reaches. Depletions result from such items as reservoir evaporation, channel losses, irrigation diversions and domestic and industrial applications. Gains are from such items as tributary, flood inflow and underground contributions. In general, all natural or man-made activities on the river are considered in a routing study. The routing begins at the upper reach and descends to the next lower reach at which net incoming flow needs to be estimated. In doing so we have to preserve mass balance and all relevant hydrological conditions. The "times" involved are <u>whole</u> periods, typically months.

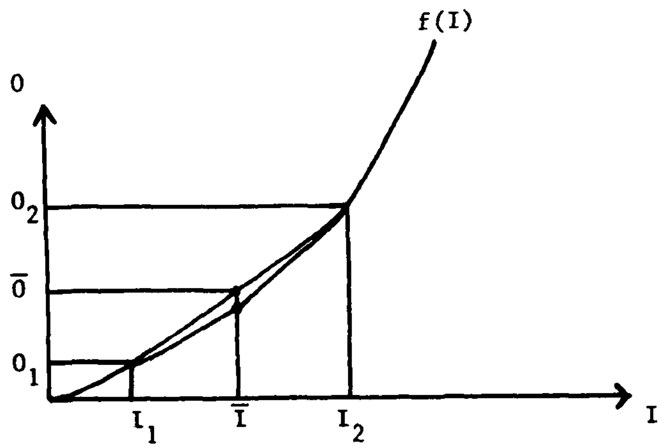
To our knowledge, the methods applied to the Pecos River to achieve monthly routing studies is to estimate losses and gains reach by reach and then for each reach add all the relevant components, keeping mass balance in each reach and between reaches. In all the routing studies we have seen there is a problem in the flood inflow computed values. Usually the flood inflow is computed as a residual from the mass balance equations after estimating all outflows and inflows. The difference between the sum of out-flows and the sum of inflows is the flood inflow, sometimes termed residual. The outflows here are evaporations, diversions for irrigation, channel losses, depletions by pumps, outflow at the end of the reach, and reservoir change in storage. The inflows are inflow to the reach, return from irrigation, and base gains in terms of underground contributions or tributarires.

The flood inflow is a non-negative quantity and by using the usual methods of doing a routing study sometimes we have negative flood inflow, which violates hydrological common sense and disrupts the mass balance equations as in [7] and [11].

We think if the components of the usual way of doing routing study are estimated on a seasonal basis instead of monthly and routed seasonally, there might not be negative flood inflows because the seasonal representation will give a better time matchup of flows including reservoir detentions and releases than averaged monthly data.

Another cause of negative flood inflow is underestimation of the outflow, since the monthly data are an average of the daily flows and the inflow-outflow functions in a reach are convex. Because of convexity, an outflow resulting from an average inflow is less than the average of outflows as seen in Figure 2.

Figure 2



I_1 = first inflow $\qquad O_1 = f(I_1)$

I_2 = second inflow $\qquad O_2 = f(I_2)$

$\bar{I}$ = average of $I_1, I_2 = \frac{I_1+I_2}{2} \qquad \bar{O} = \frac{O_1 + O_2}{2}$

We see that $f\left(\frac{I_1 + I_2}{2}\right) < \bar{0}$ and that this follows from the convexity.

We also note that underestimation of channel losses or overestimation of inflows may result in negative flood inflow. All these should be considered after removing the effect of lag. The above drawbacks in the usual methods of doing routing studies stimulated the following new methodology.

The new methodology of doing monthly routing studies consists of three parts:

(a) Hydrological adjustment: This is done in order to have a better monthly data representation instead of just average monthly data. This representation should be as close as possible to recorded data and should insure realistic sequential connection in time and reaches.

(b) Mass balance equations for all the gages on the river: routing is done with the new adjusted estimates.

(c) Yearly requirement satisfaction: that is, the new estimated monthly data should add up to yearly recorded data or be as close as possible.

This is shown in the general statement of the model in the above order.

The routing study is done simultaneously with estimating each component reach by reach using a robust statistical method, i.e. least absolute value criterion instead of sensitive least squares [4, 5, 2, 9]. To achieve the above three steps we make a least absolute value "best fit" to the gages' monthly observational sample data under realistic hydrological constraints, i.e. no negative flood inflows plus (b) and (c) on the new estimated monthly gages' readings.

Mathematically, the closeness to monthly recorded data is achieved by minimizing a heavily penalized sum of absolute deviations from the monthly recorded data (see Section 2.2). The model form is of a basic nonlinear constrained median form which will simultaneously estimate new losses, reservoir storage, spill, flood inflows, etc.

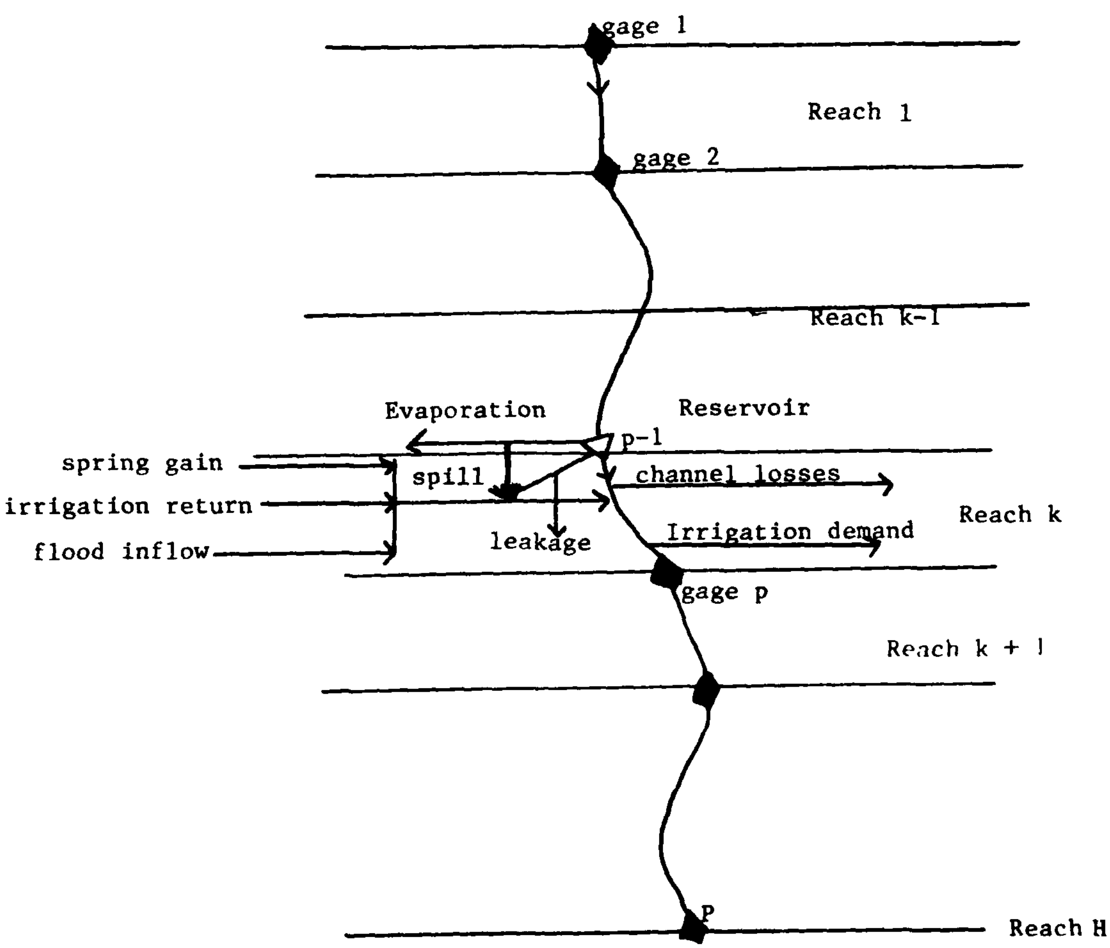
The routing process is sequential in time as well as reaches and each run develops all the monthly routings for a year simultaneously. For purposes of computation only, the nonlinear functions in the model are approximated by piece-wise linear functions in order to use a linear programming code which can accommodate the large size of the problem.

Summing up, the new routing study differs from the previous methods applied to the Pecos River in that our mathematical model automatically and objectively makes best fit adjustments to monthly data so that mass balance is preserved, so that the monthly adjusted data are as close as possible to the recorded monthly data, and so that the estimates add up as closely as possible to the yearly total.

2.1 FORMULATION OF THE MODEL

Consider a hypothetical river with H reaches, R reservoirs, P gages and G springs. The reaches are numbered beginning at the head of the river and proceeding downward. We only have irrigation demands in some of the reaches. The inflow-outflow functions are developed only for K reaches out of all the reaches, and they are monotone and convex. The fitting or best fit is done for gages in reaches where we have inflow-outflow functions; it could be done for all the gages. Figure 3 depicts the river.

Figure 3



2.2 STATEMENT OF THE MODEL

Let:

$I(i,p)$ = observed inflow at gage p in month i;

$O(i,p)$ = estimated outflow from gage p in month i using observed inflow and the inflow-outflow functions estimated reach by reach.

$\hat{f}_{ip}(I(i,p)) = O(i,p)$.

$x(i,p)$ = estimated inflow at gage p in month i;

$y(i,p)$ = estimated outflow at gage p in month i;

$w(p)$, $w(i,p)$ = weights on the goals.

Note: Inflow at gage p is the same as what leaves gage p if p is not at a reservoir or at a diversion point. Outflow at gage p is what leaves gage p minus what is lost until it reaches gage p + 1.

We can express our best fit objective and constraints as:

$$\text{Min} \quad \sum_p \sum_i y(i,p) + \sum_p \sum_i w(i,p)\,|x(i,p) - I(i,p)|$$

subject to:

(a) $\hat{f}_{ip}(x(i,p)) \quad -y(i,p) \le 0\,, \quad p = 1,\ldots,K$

(b) mass balance equations for gage p , p = 1,...,P

(c) $$\sum_{i=1}^{12} x(i,p) - \sum_{i=1}^{12} I(i,p) = 0$$

$$\sum_{i=1}^{12} y(i,p) - \sum_{i=1}^{12} O(i,p) = 0\,, \quad p = 1,\ldots,K$$

To insure feasibility always, one can goal the yearly requirements instead of having them hold exactly as in (c). The $\hat{f}_i$'s are convex functions and the spill, leakage and evaporation can be expressed or approximated by convex function. Hence we have a convex programming model. This can be solved by any nonlinear code. In our case we have used piecewise linear approximation and a linear programming code of Ali and Kennington [1].

3.1 PECOS RIVER CASE

The map in Figure 1 shows the area from Alamogordo to the New Mexico/ Texas border. We developed the routing study sketch in Figure 4 from it. We have four reaches:

(1) Alamogordo reservoir to Artesia;

(2) Artesia to McMillan dam;

(3) McMillan dam to Carlsbad;

(4) Carlsbad to state line.

We also have four reservoirs:

(1) Alamogordo reservoir of capacity 132.2 thousand acre feet;

(2) McMillan reservoir of capacity 38.6 thousand acre feet;

(3) Avalon reservoir of capacity 6.0 thousand acre feet;

(4) Red Bluff reservoir 8 miles south of the state line of capacity 310 thousand acre feet.

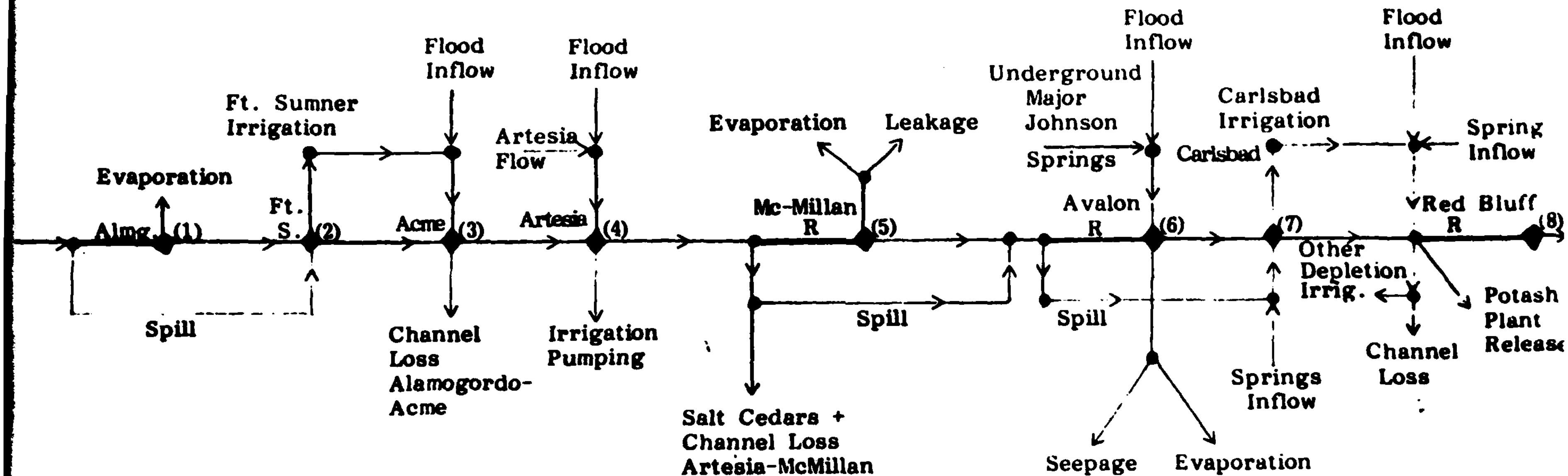
We have eight flow gages at:

(1) Alamogordo dam

(2) Fort Sumner project

FIGURE 4

Routing Representation



Almg. - Alamogordo

Ft.S. - Fort Sumner

(3) Acme

(4) Artesia

(5) McMillan dam

(6) Avalon dam

(7) Carlsbad

(8) Red Bluff.

The sketch of the river routing network is in Figure 4. The inflow-outflow functions are developed for reach two and four and a part of reach one from Alamogordo to Acme. Losses from Acme to Artesia are considered implicitly in this model. For reach three a loss of 7% is assumed for the Carlsbad main canal, i.e. an outflow of 93%. All the water for this reach is released through that canal.

The inflow for the part considered from reach one is from the gage past Sumner; for reach two it is from the Artesia gage; for reach four it is from the Carlsbad gage. The outflow for these functions is measured at Acme gage, McMillan gage, and Red Bluff gage respectively. These functions are monotone and convex.

To apply the model in Section 2.2 let us define the following variables and parameters for each year:

$x(i,p)$, $y(i,p)$, $I(i,p)$, $O(i,p)$, $w(i,p)$, $w(p)$ are as defined in section (2.2).

$K(i,r)$ = leakage from reservoir r in month i

$R(i,r)$ = release from reservoir r in month i

$S(i,r)$ = spill from reservoir r in month i

$E(i,r)$ = evaporation from reservoir r in month i

$C(r)$ = capacity or reservoir r

$C(i,r)$ = storage at end of month i in reservoir r

$F(i,h)$ = flood inflow in reach h in month i

$P(i,h)$ = pumpage from the river in reach h in month i

$G(i,h)$ = spring gains in reach h in month i

$D(i,h)$ = irrigation demand in reach h in month i

$B(i,h)$ = irrigation return in reach h in month i

$A(i)$ = Artesian gain in month i

$S_r(.)$ = spill function in reservoir r

$K_r(.)$ = leakage function in reservoir r

V_t = auxiliary parameters for ranking priority in the objective, $V_{t-1} < V_t$, $t = 1,\ldots,7$

$E_r(.)$ = evaporation function in reservoir r; all are functions of content.

$\hat{f}_{ip}(.)$ = base inflow-outflow function in month i from gage p to gage p+1

$Z(i,r)$ = auxiliary variable for content in order to compute spill in reservoir r in month i;

$T(i,r)$ = auxiliary variable for content in order to compute evaporation or leakage in reservoir r in month i;

$N(i,r)$ = auxiliary variables for content in order to compute leakage in reservoir r in month i;

$M(i,r)$ = auxiliary variables for content in order to compute evaporation in reservoir r in month i;

The last auxiliary variables to compute spill, leakage, and evaporation for reservoirs.

The gages for which we have inflow-outflow functions are (1) to (1), (2) to (3), (4) to (5), (7) to (8). (1) to (1) is done for completion and accuracy.

3.2 THE MODEL SPECIFIED FOR THE PECOS

$$\text{Min} \sum_p \sum_i y(i,p) + \sum_p \sum_i w(i,p)\,(P(i,p) + N(i,p)) + \sum_p w(p)(P(p) + N(p))$$
$$+ \sum_p w(p)\left(\bar{P}(p) + \bar{N}(p)\right) + \sum_{r=1}^{3}\sum_{i=1}^{12}\sum_{t=1}^{7} V_t N(i,r,t) + \sum_{r=1}^{3}\sum_{i=1}^{12}\sum_{t=1}^{7} V_t M(i,r,t)$$

subject to:

(a) (1) $x(i,1) \qquad -y(i,1) \le 0$

(2) $\hat{f}_{i2}(x(i,2)) \qquad -y(i,2) \le 0$

(3) $\hat{f}_{i4}(x(i,4)) \qquad -y(i,4) \le 0$

(4) $\hat{f}_{i7}(x(i,7)) \qquad -y(i,7) \le 0$

(b) (1) $x(i,1) - R(i,1) - Z(i,1) + C(i-1,1) = 0$

$Z(i,1)-S(i,1)-E(i,1)-C(i,1) = 0$

$S_1(Z(i,1))-S(i,1) = 0$

$Z(i,1)-S(i,1)-T(i,1) = 0$

$\frac{1}{2}T(i,1)+\frac{1}{2}C(i-1,1)-M(i,1) = 0$

$-E(i,1) \qquad +E_1(M(i,1)) = 0$

(2) $R(i,1) \quad +S(i,1) \quad -R(i,2) \quad +B(i,1) \quad + \quad x(i,2) = 0$

(3) $y(i,2)-x(i,3) = 0$

(4) $x(i,3)+A(i,1)+F(i,1)-P(i,1)-x(i,4) = 0$

(5) $y(i,4)+F(i,2)-R(i,2)-Z(i,2)+C(i-1,2) = 0$

$Z(i,2)-S(i,2)-E(i,2)-K(i,2)-C(i,2) = 0$

$S_2(Z(i,2))-S(i,2) = 0$

$Z(i,2) \quad - S(i,2) \quad -T(i,2) = 0$

$\frac{1}{2}T(i,2)+\frac{1}{2}C(i-1,2)-N(i,2) = 0$

$-K(i,2)+K_2(N(i,2)) = 0$

$-K(i,2)+N(i,2)-M(i,2) = 0$

$-E(i,2) \qquad +E_2(M(i,2)) = 0$

(6) $R(i,2)+S(i,2)+K(i,2)+G(i,3)+F(i,3)-R(i,3)-Z(i,3)+C(i-1,3) = 0$

$Z(i,3)-S(i,3)-E(i,3)-K(i,3)-C(i,3) = 0$

$S_3(Z(i,3))-S(i,3) = 0$

$Z(i,3)-S(i,3)-T(i,3) = 0$

$\frac{1}{2}C(i-1,3)+\frac{1}{2}T(i,3)-N(i,3) = 0$

$-K(i,3)+K_3(N(i,3)) = 0$

$-K(i,3)+N(i,3)-M(i,3) = 0$

$-E(i,3) + E_3(M(i,3)) = 0$

(7) $R(i,3) + S(i,3)+K(i,3) - x(i,7) = 0$

(8) $y(i,7) + F(i,4) - x(i,8) + G(i,4) = 0$

$R(i,1) - D(i,1) \geq 0$

(c) monthly goals

$x(i,p) - I(i,p) + P(i,p) + N(i,p) = 0$

yearly requirements

$$\sum_{i=1}^{12} x(i,p) - \sum_{i=1}^{12} I(i,p) + P(p) + N(p) = 0$$

$$\sum_{i=1}^{12} y(i,p) - \sum_{i=1}^{12} O(i,p) + \tilde{P}(p) - \bar{N}(p) = 0$$

$i = 1 \ldots 12, \quad p = 1, 2, 4, 7$

3.3 OTHER COMPUTATIONAL DETAIL

3.3.1 Spill:

Spill is the maximum of the water available in a month in the reservoir after irrigation release minus the reservoir capacity. Thus the spill in month i from reservoir r is:

(3.3.1.1) $S_r(Z(i,r)) = S(i,r) = \max(Z(i,r) - C(r), 0)$

(3.3.1.2) $S_r(Z(i,r)) = S(i,r) = \frac{1}{2}[\,|Z(i,r) - C(r)| + Z(i,r) - C(r)]$

To allow the model to compute spill automatically, we broke the Z(i,r) into two parts, a spill part and a non-spill part. We assigned priority coefficients in the objective to assure that the non-spill part comes into the optimal solution first. The spill part will not come in unless the non-spill is equal to the reservoir capacity. Thus we will add the following to the model:

$$\text{objective} + \sum_{r=1}^{3} \sum_{i=1}^{12} \sum_{t=1}^{2} V_t Z(i,r,t)$$

and

$$\begin{aligned} Z(i,r) \;-\; Z(i,r,t) - Z(i,r,2) &\leq 0 \\ Z(i,r,t) &\leq c(r), \end{aligned}$$

for r = 1, 2, 3 and i = 1,...,12.

An optimal solution for the model will give Z*(i,r,2) as spill, i.e. S(i,r) = Z*(i,r,2) for month i, reservoir r. We note that after irrigation release comes spill, then leakage, and then evaporation.

3.3.2 Leakage

Since the Alamogordo reservoir doesn't leak and the Red Bluff reservoir is south of the state line, no content-leakage relationship is developed for them.

A content-leakage relationship is developed for McMillan reservoir and Avalon reservoir using Tables 1 and 2, and they are shown in Figures 5 and 6. The relationships are then piecewise linearized and they are of the following form:

1. McMillan

(3.3.2.1) $$K(i,2) = 1.68N(i,2,1) + 0.32N(i,2,2) + 0.2N(i,2,3) + 0.13N(i,2,4) + 0.13N(i,2,5) + 0.12N(i,2,6) + 0.11N(i,2,7)$$

with $0 \leq N(i,2,5) \leq 5,\ t = 1,\ldots,6,\quad 0 \leq N(i,2,7) \leq 8.5$

$i = 1,\ldots,12$

and $\sum_{t=1}^{7} N(i,2,t) = N(i,2)$

2. Avalon

(3.3.2.2) $$K(i,3) = 0.5N(i,3,1) + 0.6N(i,3,2) + 0.4N(i,3,3) + 0.4N(i,3,4) + 0.4N(i,3,5) + 0.3N(i,3,6)$$

with $0 \leq N(i,3,t) \leq 1,\quad t = 1,\ldots,6$

and $\sum_{t=1}^{6} N(i,3,t) = N(i,3)$

We assign priority coefficients in the objective function to assure that lower parts of the content in the reservoir come in first in an optimal solution.

3.3.3 Evaporation

To allow the model to compute evaporation, we express surface area as a function of content. Then we use the monthly evaporation coefficients per unit surface to compute monthly evaporations because the available evaporation coefficients are in terms of surface area.

For Alamogordo, the data in Table 3 is used to develop content-surface area relationship as shown in Figure 7. The monthly evaporations are given by:

(3.3.3.1) $$E(i,1) = e_{1iy}[S(i,1)] = e_{1iy}[\,0.0525M(i,1,1) + 0.038M(i,1,2) + 0.0346M(i,1,3) + 0.0338M(i,1,4) + 0.02586M(i,1,5) + 0.0318M(i,1,6) + 0.0251M(i,1,7)]$$

with $0 \leq M(i,1,t) \leq 20,\ t = 1,\ldots,6,\quad 0 \leq M(i,1,7) \leq 12.2$

$$\text{and} \quad \sum_{t=1}^{7} M(i,1,t) = M(i,1) \text{ where } e_{1iy} = \text{monthly evaporation}$$

coefficient for month i, year y and reservoir 1.

The data in Table 4 is used for McMillan to develop the content-surface area relationships shown in Figure 8. The monthly evaporation is given by:

$$(3.3.3.2) \quad E(i,2) = e_{2iy}[S(i,2)] = e_{2iy}[0.478M(i,2,1) + 0.126M(i,2,2) + 0.106M(i,2,3) + 0.074M(i,2,4) + 0.078M(i,2,5) + 0.074M(i,2,6) + 0.083M(i,2,7)]$$

$$\text{with } 0 \leq M(i,2,t) \leq 5,\ t = 1,\ldots,6, \quad 0 \leq M(i,2,7) \leq 8.5$$

$$\text{and} \quad \sum_{t=1}^{7} M(i,2,t) = M(i,2)$$

The data in Table 5 for Avalon is used to develop the content-surface area relationship shown in Figure 9. The monthly evaporation is given by:

$$(3.3.3.3) \quad E(i,3) = e_{31y}[S(i,3)] = e_{31y}[0.425M(i,3,1) + 0.195M(i,3,2) + 0.1M(i,3,3) + 0.05M(i,3,4) + 0.135M(i,3,5) + 0.08M(i,3,6)]$$

$$\text{with } 0 \leq M(i,3,t) \leq 1,\ t = 1,\ldots,6$$

$$\text{and} \quad \sum_{t=1}^{6} M(i,3,5) = M(i,3)$$

Red Bluff evaporation is not considered in the model since Red Bluff is 8 miles south of the state line. Again we assign priority coefficients in the objective function to assure that the lower part of the content comes in first in an optimal solution.

3.3.4 Flood Inflow

Flood inflow is computed as a residual in the mass balance equations. They are constrained to be non-negative, which is assured because the model

automatically adjusts all other estimated quantities to conform to reality.

3.3.5 Base Inflow-Outflow Functions

The base inflow-outflow functions are developed using a least absolute value criterion. They are then approximated in a piecewise linear fashion. The following functions are used for the first subreach (Alamogordo to Acme):

For January, February, December, and November,

$$\hat{f}_{i2}(x(i,2)) = 0.0x(i,2,1) + 0.915x(i,2,2) + 0.983x(i,2,3) + 0.989x(i,2,4) + 0.991x(i,2,5) + 0.993x(i,2,6) + 0.995x(i,2,7) + x(i,2,8) \tag{3.3.5.1}$$

$$0 \le x(i,2,1) \le 1,\ 0 \le x(i,2,2) \le 9,\ 0 \le x(i,2,j) \le 10,\ j = 3,\dots,7$$

$$0 \le x(i,2,8) \le 500$$

$$\sum_{j=1}^{8} x(i,2,j) = x(i,2),\ i = 1,2,11,12$$

For March,

$$\hat{f}_{32}(x,3,2)) = 0.0x(3,2,1) + 0.978x(3,2,2) + 0.993x(3,2,3) + 0.996x(3,2,4) + 0.997x(3,2,5) + 0.998x(3,2,6) + 0.998x(3,2,7) + x(3,2,8) \tag{3.3.5.2}$$

$$0 \le x(3,2,1) \le 1,\ 0 \le x(3,2,2) \le 9,\ 0 \le x(3,2,j) \le 10,\ j = 3,\dots,7$$

$$0 \le x(3,2,8) \le 500$$

$$\sum_{j=1}^{8} x(3,2,j) = x(3,2)$$

For July, August, and September,

$$(3.3.5.6)\quad \hat{f}_{i2}(x(i,2)) = 0.0x(i,2,1) + 0.825x(i,2,2) + 0.943x(i,2,3) + 0.943x(i,2,4) + 0.962x(i,2,5) + 0.97x(i,2,6) + 0.976x(i,2,7) + x(i,2,8)$$

$$0 \leq x(i,2,1) \leq 1,\ 0 \leq x(i,2,2) \leq 9,\ 0 \leq x(i,2,j) \leq 10,\ j = 3,\ldots,7$$

$$0 \leq x(i,8,1) \leq 500$$

$$\sum_{j=1}^{8} x(i,2,j) = x(i,2) \quad i = 7, 8, 9$$

For October,

$$(3.3.5.7)\quad \hat{f}_{102}(x(i,2)) = 0.0x(10,2,1) + 0.738x(10,2,2) + 0.864x(10,2,3) + 0.896x(10,2,4) + 0.913x(10,2,5) + 0.923x(10,2,6) + 0.931x(10,2,7) + x(10,2,8)$$

$$0 \leq x(10,2,2) \leq 1,\ 0 \leq x(10,2,2) \leq 9,\ 0 \leq x(10,2,j) \leq 10,\ j = 1,\ldots,7$$

$$0 \leq x(6,2,8) \leq 500$$

$$\sum_{j=1}^{8} x(10,2,j) = x(10,2)$$

In the second reach, Artesia to McMillan, one base inflow-outflow relation was used for all months:

$$(3.3.5.8)\quad \hat{f}_{i4}(x(8,4)) = 0.0x(i,4,1) + 0.661x(i,4,2) + 0.787x(i,4,3) + 0.827x(i,4,4) + 0.849x(i,4,5) + 0.863x(i,4,6) + 0.87x(i,4,7) + x(i,4,8)$$

$$0 \leq x(i,4,1) \leq 1,\ 0 \leq x(i,4,2) \leq 9,\ 0 \leq x(i,4,j) \leq 10,\ j = 3,\ldots,7$$

$$0 \leq x(i,4,8) \leq 500$$

$$\sum_{j=1}^{8} x(i,4,j) = x(i,4\ )\ ,\ i = 1,\ldots,12.$$

For April,

$$(3.3.5.3)\quad \hat{f}_{42}(x(4,2)) = 0.0x(4,2,1) + 0.89x(4,2,2) + 0.954x(4,2,3) + 0.969x(4,2,4) + 0.975x(4,2,5) + 0.979x(4,2,6) + 0.982x(4,2,7) + x(4,2,8)$$

$$0 \leq x(4,2,1) \leq 1, \; 0 \leq x(4,2,2) \leq 9, \; 0 \leq x(4,2,j) \leq 10, \; j = 3,\ldots,7$$

$$0 \leq x(4,2,8) \leq 500$$

$$\sum_{j=1}^{8} x(4,2,j) = x(4,2)$$

For May,

$$(3.3.5.4)\quad \hat{f}_{52}(x(5,2)) = 0.0x(5,2,1) + 0.819x(5,2,2) + 0.919x(5,2,3) + 0.941x(5,2,4) + 0.952x(5,2,5) + 0.959x(5,2,6) + 0.964x(5,2,7) + x(5,2,8)$$

$$0 \leq x(4,2,1) \leq 1, \; 0 \leq x(5,2,2) \leq 9, \; 0 \leq x(5,2,j) \leq 10, \; j = 3,\ldots,7$$

$$0 \leq x(5,2,8) \leq 500$$

$$\sum_{j=1}^{8} x(5,2,j) - x(5,2)$$

For June,

$$(3.3.5.5)\quad \hat{f}_{62}(x(6,2)) = 0.0x(6,2,1) + 0.699x(6,2,2) + 0.823x(6,2,3) + 0.862x(6,2,4) + 0.882x(6,2,5) + 0.895x(6,2,6) + 0.904x(6,2,7) + x(6,2,8)$$

$$0 \leq x(6,2,1) \leq 1, \; 0 \leq x(6,2,2) \leq 9, \; 0 \leq x(6,2,j) \leq 10, \; j = 3,\ldots,7$$

$$0 \leq x(6,2,8) \leq 500$$

$$\sum_{j=1}^{8} x(6,2,j) = x(6,2)$$

For the third reach we take as an outflow 93% of inflow. The reason for this is explained in Section (3,1), paragraph 2.

In the fourth reach, Carlsbad to New Mexico/Texas state line, a yearly inflow-outflow relationship is developed. Then the monthly distribution of the losses is used to get the outflow on a monthly basis. This method is used instead of monthly base inflow-outflow functions. The percentage distribution of losses over the various months is presented in [7], Appendix 20.

Naturally, appropriate software, e.g. a matrix generator, was developed to take data for a year and put it into the form for solution by our linear programming software, SMULP. With these facilities, routing studies were run for the 29 years from 1919 to 1947.

RESULTS AND CONCLUSIONS

Prior to the development of the median routing study method, two routing studies had been done for the Pecos River. The first one is in [7], referred to here as SD109. The second one is [11], referred to as RBD. Table 6 is a summary of the results of the three routing studies. For each study, two columns show the three-year moving average outflow of the border and the corresponding three-year moving average of index inflow. The index inflow for RBD and SD109 is the sum of all the flood inflows in the four reaches plus the routed inflow at Alamogordo reservoir. For the median routing study the index inflow is the sum of all flood inflows in all four reaches plus the adjusted inflow at the Alamogordo reservoir.

The reader should be cautioned in examining the graphs of the overall inflow-outflow relationship shown in Figure 5 for the three routing studies that they are not directly comparable due to the fact that each routing study has a different index inflow.

In the SD109 routing studies, negative flood inflows are set to zero. In the RBD, the unreal negative flood inflow quantities are employed in their routing study calculations as if the mass balance equations applied to non-real possibilities. The water estimates in SD109 are deficient in that no objective statistical estimation method is provided to justify setting estimates at zero.

The RBD negatives, which ignore hydrological and physical reality, effectively subtract water from that which must have been in the river to produce the observed time sequential data in a year. Thus both these studies estimate less water in the river than does our median routing study.

Quantitative comparison of the delivery of the three studies has been made in terms of average outflow at the border over the 29 years for which the routing studies were performed. The distortions produced by the ad hoc method in SD109 and the negatives of the RDB method have the median routing study averaging 7% higher outflow than SD109 and 32% higher than the RBD.

Such substantial differences should demonstrate the need to employ the more complicated type of method we have developed to insure valid estimates in routing studies. Our software is available to anyone, subject only to reimbursing us for copying, materials and transportation expenses.

Because of time pressures we did not explore modeling refinements which might greatly improve the computational speed and capability to handle longer problems. We intend in further research to try to go from a general linear programming model to a (generalized or pure) network format which can provide a two orders of magnitude improvement in speed and size of the problem which can be handled.

Another direction of extension of our results might be in more explicit introduction ot stochastic elements into routing studies, e.g. the use of probability distributions for flood inflow computations, satisfaction of irrigation demands, and so on.

TABLE 1

McMillan Reservoir

1947 Condition

Gage Height, Capacity, Area, Leakage Relation

Gage Height	Capacity	Area	Leakage	Gage Height	Capacity	Area	Leakage
12.0	0	.4	0	19.2	10.0	3.0	10.0
12.2	.1	.4	0	19.4	10.6	3.1	10.1
12.4	.1	.4	0	19.6	11.2	3.1	10.2
12.6	.2	.4	.1	19.8	11.8	3.2	10.4
12.8	.2	.4	.1	20.0	12.4	3.3	10.5
13.0	.3	.5	.1	20.2	13.0	3.4	10.6
13.2	.3	.5	.2	20.4	13.6	3.4	10.7
13.4	.4	.5	.2	20.6	14.3	3.5	10.8
13.6	.5	.6	.3	20.8	15.0	3.6	11.0
13.8	.5	.6	.4	21.0	15.7	3.6	11.1
14.0	.6	.6	.5	21.2	16.5	3.7	11.2
14.2	.6	.7	.6	21.4	17.2	3.7	11.3
14.4	.7	.7	.7	21.6	18.0	3.8	11.4
14.6	.8	.8	.8	21.8	18.8	3.8	11.5
14.8	.8	.8	1.0	22.0	19.6	3.9	11.6
15.0	.9	.9	1.1	22.2	20.4	3.9	11.7
15.2	1.0	1.0	1.3	22.4	21.2	4.0	11.8
15.4	1.3	1.2	1.6	22.6	22.0	4.1	11.9
15.6	1.6	1.3	2.0	22.8	22.8	4.1	12.0
15.8	1.9	1.5	3.0	23.0	23.6	4.2	12.2
16.0	2.2	1.6	4.0	23.2	24.4	4.3	12.3
16.2	2.6	1.8	5.1	23.4	25.2	4.4	12.4
16.4	3.0	1.9	6.2	23.6	26.0	4.4	12.5
16.6	3.3	2.0	7.1	23.8	27.0	4.5	12.6
16.8	3.7	2.1	7.6	24.0	28.0	4.6	12.7
17.0	4.0	2.2	7.9	24.2	29.0	4.7	12.8
17.2	4.5	2.3	8.2	24.4	30.0	4.8	12.9
17.4	5.0	2.4	8.4	24.6	31.0	4.9	13.0
17.6	5.5	2.5	8.6	24.8	32.0	5.0	13.1
17.8	6.0	2.5	8.8	25.0	33.0	5.1	13.2
18.0	6.5	2.6	9.0	25.2	34.0	5.1	13.3
18.2	7.0	2.7	9.2	25.4	35.0	5.2	13.4
18.4	7.5	2.8	9.4	25.6	36.0	5.3	13.5
18.6	8.0	2.9	9.5	25.8	37.0	5.4	13.6
18.8	8.6	2.9	9.7	26.0	38.0	5.4	13.7
19.0	9.3	3.0	9.8	26.1	38.6	5.5	13.8

UNITS: Gage Height - feet
Capacity - 1000 acre-feet
Surface Area - 1000 acres
Leakage - 1000 acre-feet per month

TABLE 2

AVALON RESERVOIR
1947 Condition

Capacity - Area - Leakage

Cap. 1000 AF	Area 1000 Acres	Leakage 1000 AF Month	Cap. 1000 AF	Area 1000 Acres	Leakage 1000 AF Month
0	0.1	0	3.1	0.8	1.6
0.1	0.1	0	3.2	0.8	1.6
0.2	0.2	0	3.3	0.8	1.6
0.3	0.2	0.1	3.4	0.8	1.7
0.4	0.3	0.1	3.5	0.8	1.7
0.5	0.3	0.1	3.6	0.8	1.8
0.6	0.3	0.2	3.7	0.8	1.8
0.7	0.3	0.4	3.8	0.8	1.9
0.8	0.4	0.4	3.9	0.8	1.9
0.9	0.4	0.5	4.0	0.8	1.9
1.0	0.4	0.5	4.1	0.8	2.0
1.1	0.5	0.6	4.2	0.8	2.0
1.2	0.5	0.7	4.3	0.9	2.0
1.3	0.5	0.7	4.4	0.9	2.1
1.4	0.5	0.8	4.5	0.9	2.1
1.5	0.5	0.8	4.6	0.9	2.1
1.6	0.6	0.9	4.7	0.9	2.2
1.7	0.6	1.0	4.8	0.9	2.2
1.8	0.6	1.0	4.9	0.9	2.2
1.9	0.6	1.0	5.0	0.9	2.3
2.0	0.6	1.1	5.1	0.9	2.3
2.1	0.6	1.2	5.2	0.9	2.3
2.2	0.7	1.2	5.3	0.9	2.4
2.3	0.7	1.2	5.4	1.0	2.4
2.4	0.7	1.3	5.5	1.0	2.4
2.5	0.7	1.3	5.6	1.0	2.5
2.6	0.7	1.4	5.7	1.0	2.5
2.7	0.7	1.4	5.8	1.0	2.5
2.8	0.7	1.4	5.9	1.0	2.6
2.9	0.7	1.5	6.0	1.0	2.6
3.0	0.7	1.5			

TABLE 3

Alamogordo Reservoir

AREA - CAPACITY (1944)

from Corps of Engineers'1944 Sediment Survey

Elevation Feet	Capacity Acre feet	Area Acres	Elevation Feet	Capacity Acre Feet	Area Acres
4143	0	0	4238	28320	1385
4200	280	227	4240	31180	1491
4202	680	285	4242	34260	1610
4204	1230	340	4244	37600	1735
4206	1960	400	4246	41260	1865
4208	2840	450	4248	45200	2015
4210	3830	483	4250	49440	2116
4212	4910	535	4252	54000	2335
4214	6060	580	4254	58880	2515
4216	7260	620	4256	64120	2690
4218	8540	675	4258	69700	2880
4220	9900	730	4260	75590	3069
4222	11350	775	4262	81850	3250
4224	12950	830	4264	88500	3430
4226	14710	895	4266	95520	3610
4228	16590	960	4268	102940	3780
4230	18620	1022	4270	110720	3956
4232	20800	1105	4272	118910	4180
4234	23140	1195	4274	127620	4440
4236	25640	1280	4275	132170	4627

Figure 5

MCMILLAN RESERVOIR CAPACITY-LEAKAGE RELATIONSHIP

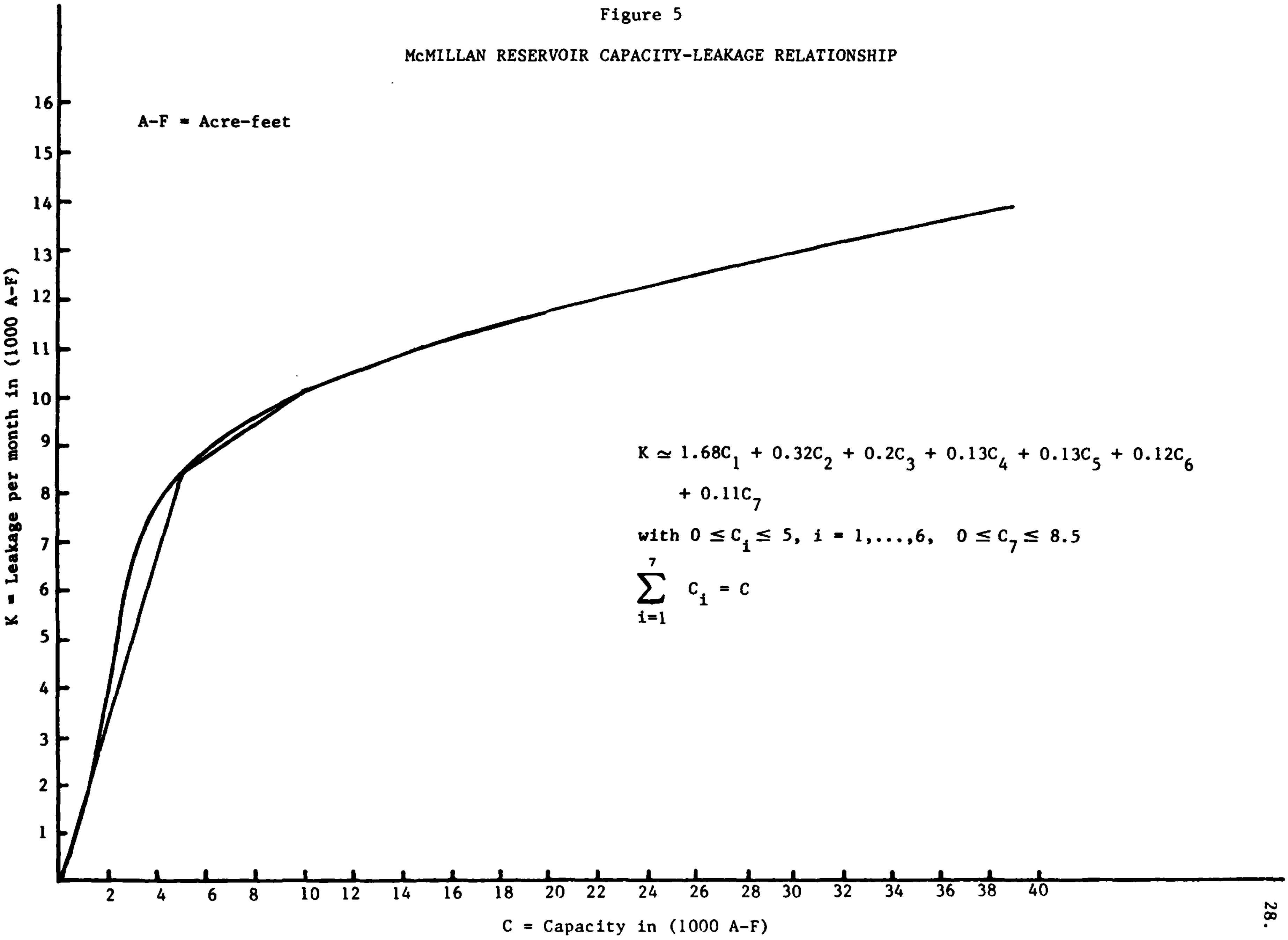


Figure 6

AVALON RESERVOIR CAPACITY-LEAKAGE RELATIONSHIP

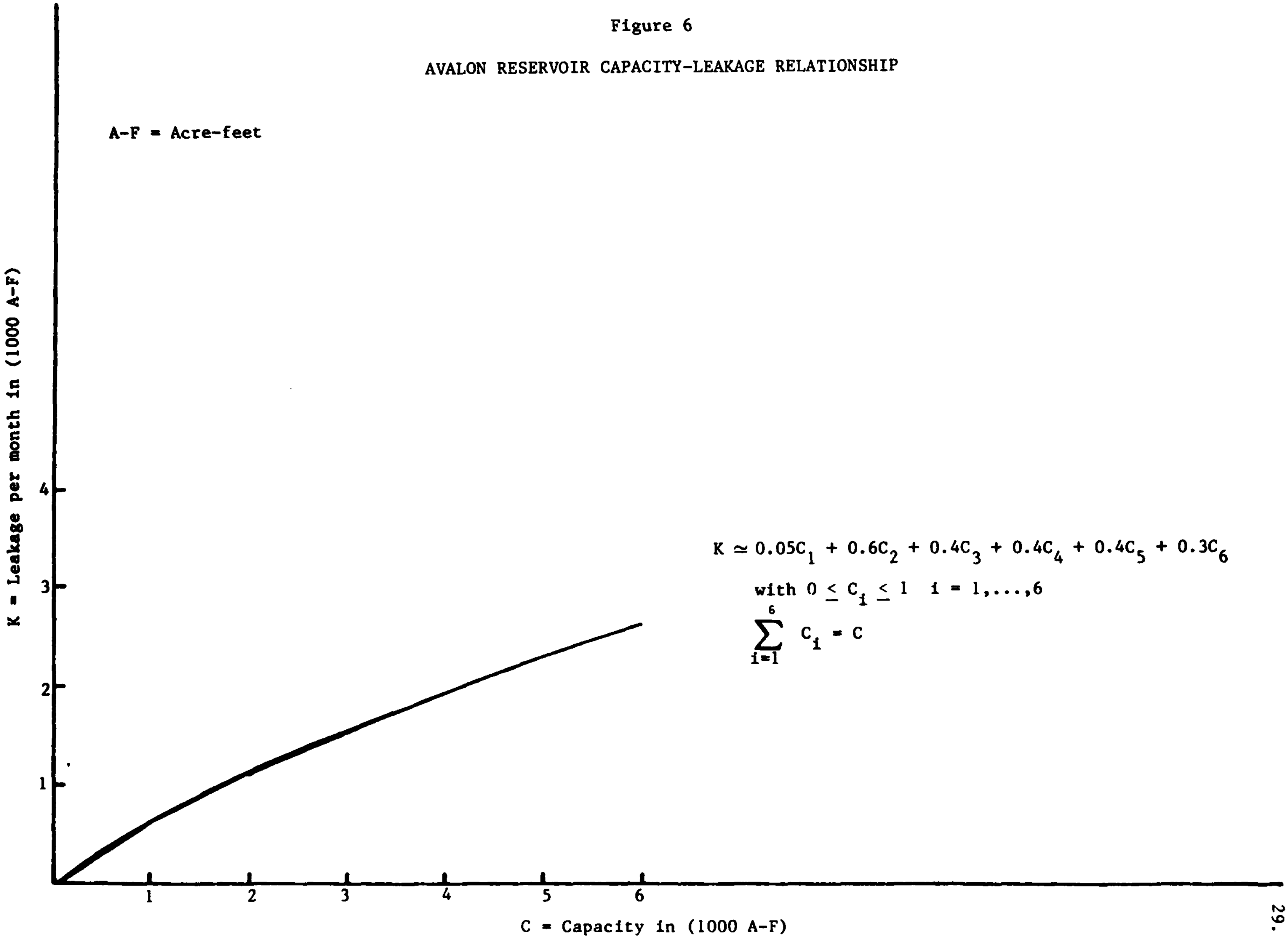
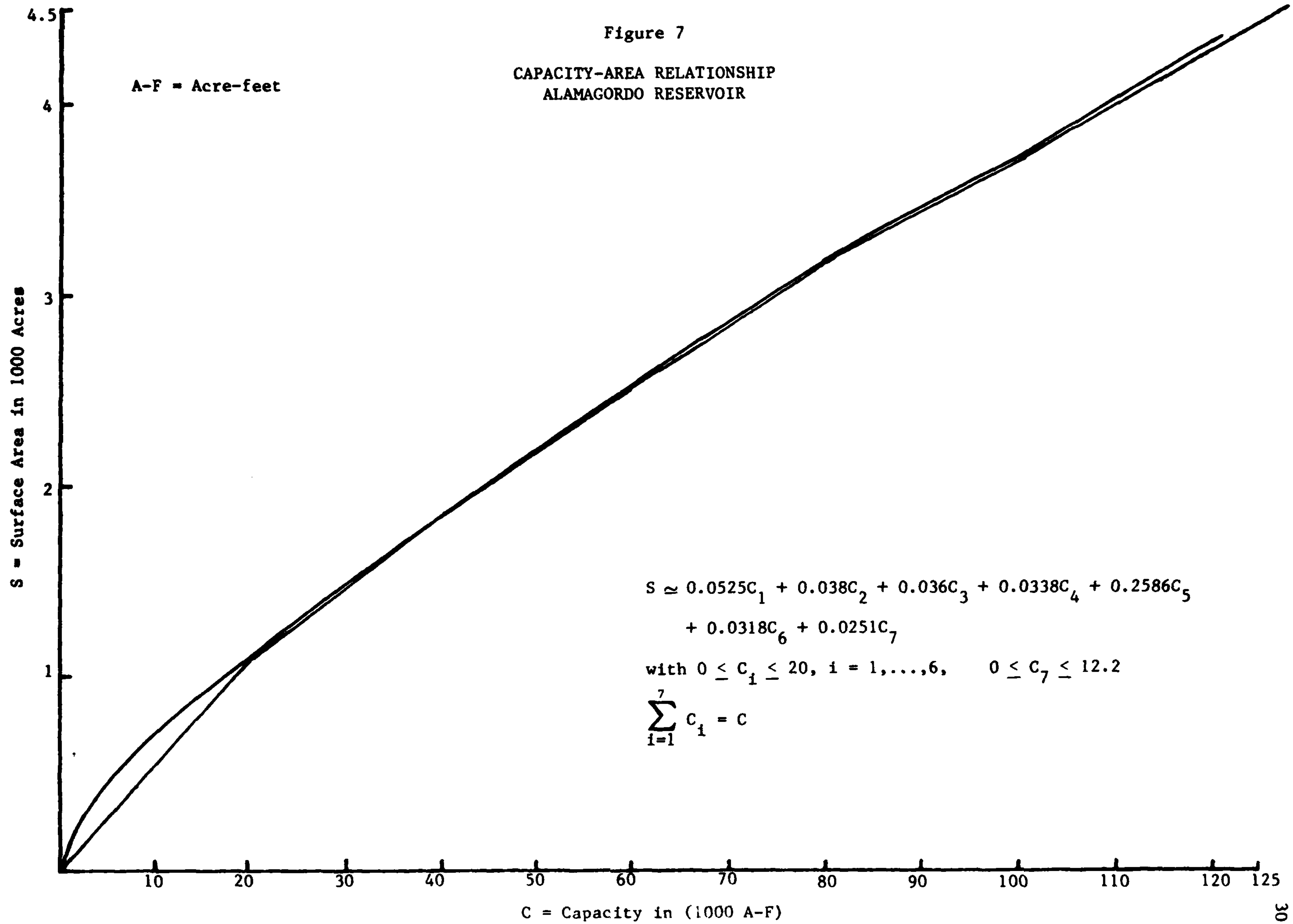


Figure 7

CAPACITY-AREA RELATIONSHIP
ALAMAGORDO RESERVOIR



$$S \simeq 0.0525C_1 + 0.038C_2 + 0.036C_3 + 0.0338C_4 + 0.2586C_5 + 0.0318C_6 + 0.0251C_7$$

with $0 \leq C_i \leq 20$, $i = 1,\ldots,6$, $\quad 0 \leq C_7 \leq 12.2$

$$\sum_{i=1}^{7} C_i = C$$

TABLE 4

McMillan Reservoir
AREA-CAPACITY (1940)

Zero Gage = 3241.6

From Senate Document 109

Gage Height (feet)	Capacity (Acre-feet)	Area (Acres)	Gage Height (feet)	Capacity (Acre-feet)	Area (Acres)
15.1	1,000	900	22.35	21,000	3,990
15.9	2,000	1,530	22.6	22,000	4,060
16.43	3,000	1,900	22.85	23,000	4,150
17.0	4,000	2,190	23.1	24,000	4,220
17.12	5,000	2,390	23.32	25,000	4,310
17.35	6,000	2,550	23.57	26,000	4,410
18.23	7,000	2,710	23.8	27,000	4,510
18.56	8,000	2,820	24.02	28,000	4,620
18.92	9,000	2,930	24.22	29,000	4,698
19.24	10,000	3,020	24.43	30,000	4,780
19.57	11,000	3,180	24.63	31,000	4,870
19.9	12,000	3,230	24.83	32,000	4,950
20.2	13,000	3,350	25.03	33,000	5,040
20.49	14,000	3,460	25.22	34,000	5,120
20.77	15,000	3,550	25.42	35,000	5,200
21.06	16,000	3,640	25.61	36,000	5,280
21.32	17,000	3,720	25.8	37,000	5,360
21.58	18,000	3,790	26.0	38,000	5,440
21.84	19,000	3,850	26.1	38,600	5,490
22.1	20,000	3,920			

TABLE 5

Avalon Reservoir

AREA-CAPACITY RELATION (1940)

Zero of gage - 3157.0

Gage Height	Capacity 1000 A.F.	Area Acres	Gage Height	Capacity 1000 A.F.	Area Acres
2.0	0	62	16.6	3.1	590
10.7	0.5	220	16.8	3.3	607
12.7	0.7	276	17.0	3.4	624
13.0	1.0	326	17.2	3.6	642
13.2	1.0	338	17.4	3.7	660
13.4	1.1	350	17.6	3.8	678
13.6	1.2	362	17.8	4.0	696
13.8	1.3	375	18.0	4.2	714
14.0	1.4	389	18.2	4.4	732
14.2	1.5	403	18.4	4.5	750
14.4	1.6	417	18.6	4.7	768
14.6	1.7	431	18.8	4.8	786
14.8	1.8	445	19.0	5.0	804
15.0	2.0	459	19.2	5.2	822
15.2	2.1	475	19.4	5.4	840
15.4	2.3	491	19.6	5.5	858
15.6	2.4	507	19.8	5.6	877
15.8	2.5	523	20.0	5.8	897
16.0	2.6	539	20.2	6.0	915
16.2	2.8	556	20.4	6.2	933
16.4	3.0	573			

The gage heights and capacities are from S.D. 109. The corresponding areas were abstracted from the 1947 River Routing Studies.

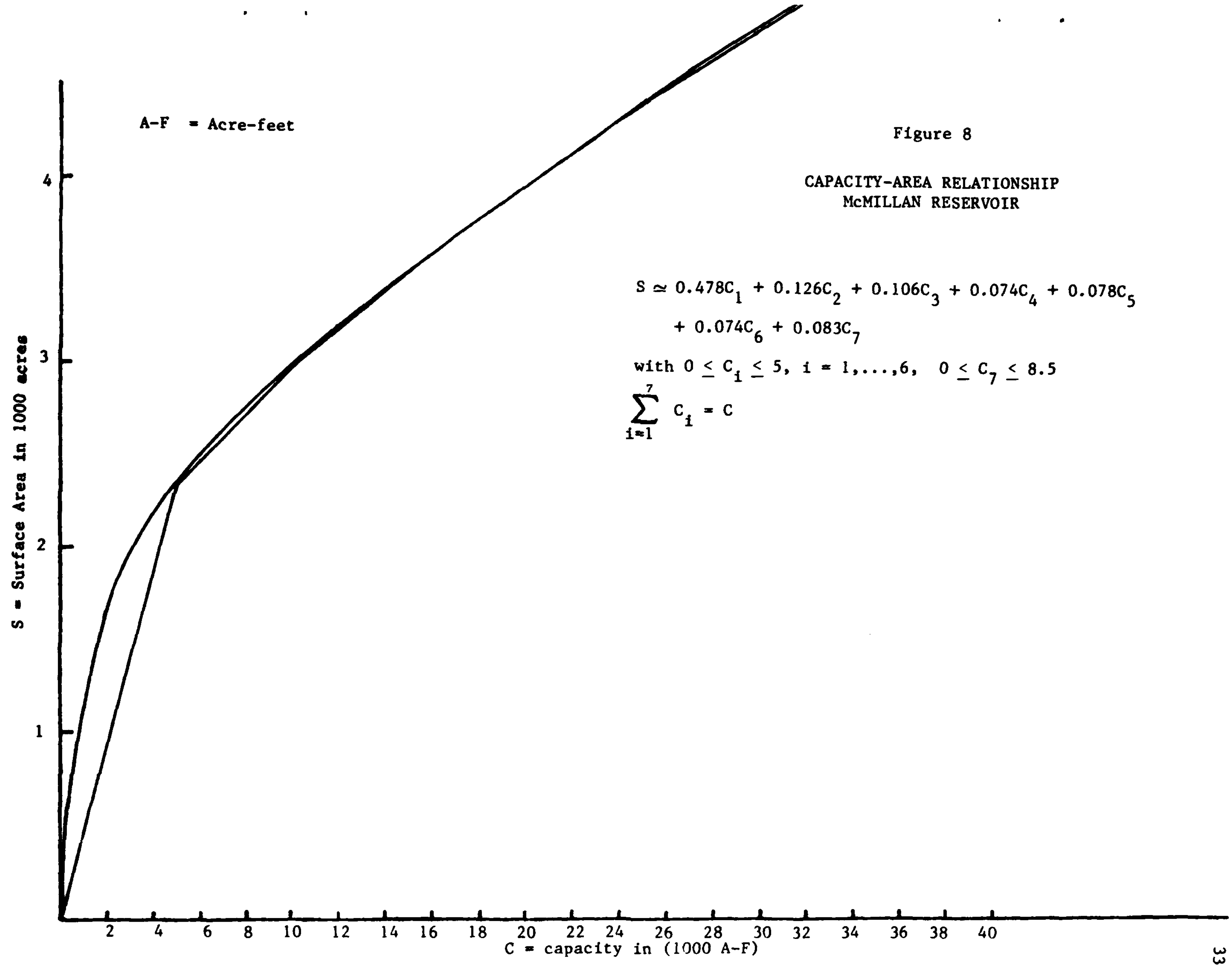


Figure 8

CAPACITY-AREA RELATIONSHIP
McMILLAN RESERVOIR

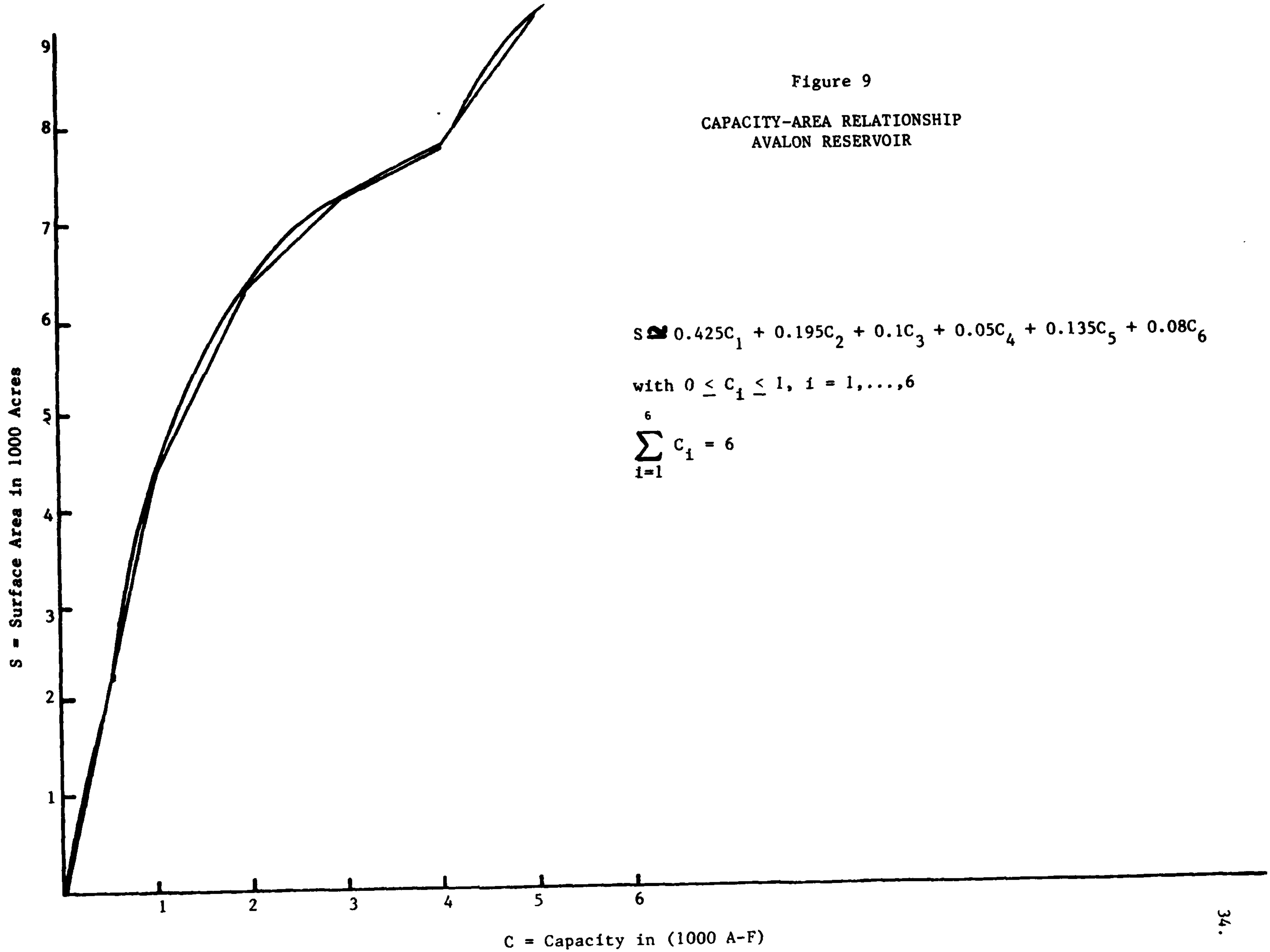
$$S \simeq 0.478C_1 + 0.126C_2 + 0.106C_3 + 0.074C_4 + 0.078C_5 + 0.074C_6 + 0.083C_7$$

with $0 \le C_i \le 5$, $i = 1,\dots,6$, $0 \le C_7 \le 8.5$

$$\sum_{i=1}^{7} C_i = C$$

Figure 9

CAPACITY-AREA RELATIONSHIP
AVALON RESERVOIR



$$S \cong 0.425C_1 + 0.195C_2 + 0.1C_3 + 0.05C_4 + 0.135C_5 + 0.08C_6$$

with $0 \le C_i \le 1$, $i = 1,\ldots,6$

$$\sum_{i=1}^{6} C_i = 6$$

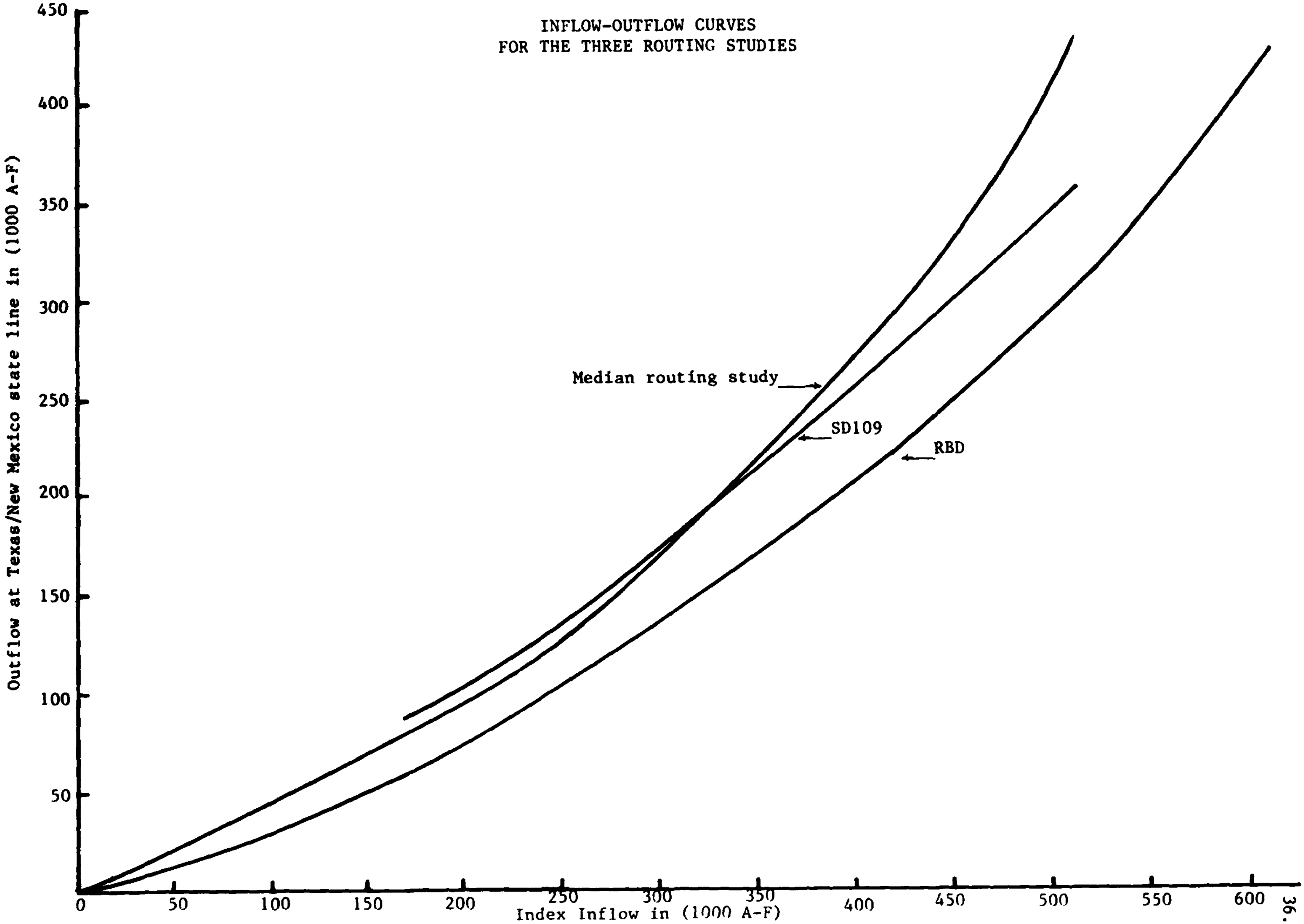
Table 6

THREE-YEAR MOVING AVERAGE 1919-1947

Year	MEDIAN INDEX INFLOW	MEDIAN OUTFLOW	SD 109 INDEX INFLOW	SD 109 OUTFLOW	RBD INDEX INFLOW	RBD OUTFLOW
1921	633.19	464.97	557.8	412.3	576.3	376.3
1922	395.83	257.8	370.3	259.9	358.8	184.6
1923	437.48	276.5	392.3	259.1	383.2	186.3
1924	331.01	197.5	268.4	156.3	293.8	116.9
1925	379.94	222.97	300.1	178.0	295.5	124.1
1926	398.76	234.82	318.7	200.6	304.4	139.3
1927	383.01	243.8	325.9	203.9	309.1	136.4
1928	339.65	215.81	307.2	187.5	320.0	137.3
1929	260.37	169.34	250.2	150.2	292.2	121.1
1930	299.04	188.99	275	168.8	296.9	125.6
1931	342.34	198.27	294.4	189.2	280.5	125.9
1932	405.15	257.67	377.2	251.7	347.2	178.2
1933	356.2	239.9	342.2	236.0	346.3	178.3
1934	296.69	219.52	292.0	191.9	306.4	152.1
1935	253.15	157.33	223.6	136.0	238.8	101.9
1936	258.06	153.02	227.4	127.8	226.7	93.5
1937	459.91	303.27	367.1	243.5	386.6	223.1
1938	456.44	304.81	388.5	253.1	409.1	232.6
1939	467.84	303.17	392.2	256.3	417.7	237.1
1940	266.99	158.12	269.0	151.1	284.3	120.8
1941	630.13	643.91	267.1	639.8	778.4	625.7
1942	705.74	728.55	859.7	732.3	861.8	710.8
1943	714.98	740.93	859.3	746.2	866.0	720.9
1944	329.06	259.96	337.4	246.2	351.5	202.5
1945	231.12	159.95	234.8	139.0	237.5	103.0
1946	236.11	143.92	201.2	121.0	215.2	83.9
1947	218.64	119.96	____	____	176.8	66.6
Average	394.4	282.0	357.2	263	376.1	214

Figure 10

INFLOW-OUTFLOW CURVES
FOR THE THREE ROUTING STUDIES



REFERENCES

1. A. Iqbal Ali, J. J. Kennington, "SMU-LP: An In-Core Primal Simplex Code for Solving Linear Programs, Version 1," Technical Report OR 80015.

2. G. Bassett, R. Koenker, "Asymptotic Theory of Least Absolute Error Regression" Journal of American Statistical Association, v. 73, No. 343, Sept. 1972, pp. 618-622.

3. P.J. Bickel, K.A. Doksum, Mathematical Statistics: Basic Ideas and Selected Topics, Hodlen-Day, Inc., 1977.

4. A. Charnes, W.W. Cooper, Management Models and Industrial Applications of Linear Programming, Vols. I and II. John Wiley and Sons, 1961, 00. 327-347.

5. A. Charnes, W.W. Cooper, R. Ferguson. "Optimal Estimation of Executive Compensation by Linear Programming" Management Science I, No. 2, January 1955, pp. 138-151.

6. V.T. Chow. Handbook of Applied Hydrology. McGraw-Hill, New York, 1964.

7. Engineering Advisory Committee to the Pecos River, "Pecos River Compact; Senate Document 109." Unpublished report.

8. J.R. Erickson, D.P. Hale, J.J. Vandertulip, C.L. Slingerland, J.C. Anderson, "Report of Review of Basic Data," submitted to Engineering Advisory Committee to the Pecos River. Unpublished report.

9. J.E. Gentle, "Lease Absolute Value Estimation: An Introduction." Commu-Statist-Simula. Computa B 6(4), 1977, pp. 313-328.

10. National Resources Planning Board, June 1942, "The Pecos River Joint Investigation of the Participating Agencies."

11. Subcommittee on Review of Basic Data, "Data Submitted to Engineering Advisory Committee, Pecos River Commission, on January 30, 1966." Unpublished report.

12. A. Prekopa, T. Szantal, "New Multivariate Gamma Distribution and its Fitting to Empirical Steam Flow Data," to appear in Journal of Water Resources/Research.

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Median Routing Study; Robust Statistical Method, Piece-wise Linear; Linear Programming; Least Absolute Value Regression; Mass Balance Equations; Inflow-Outflow Functions

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The model developed in this report is a new robust statistical methodology for achieving monthly routing studies. The model automatically and objectively makes best fit adjustment to monthly data so that mass balance is preserved, so that the monthly adjusted data are as close as possible to recorded monthly data, and so that the estimates add up as closely as possible to yearly totals. The model is of constrained median form. It simultaneously estimates new losses, reservoir storage, spills, flood inflows, etc.

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20. Abstract

For computation the model is piece-wise linearized so that LP methods can be employed. It is applied to the Pecos River between Texas and New Mexico. Comparison is made between its results and those of two other methods applied earlier to the Pecos River.

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